

# Design of Digital Circuits

## Lecture 19b: Systolic Arrays and Beyond

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ETH Zurich

Spring 2019

3 May 2019

# Bachelor's Seminar in Comp Arch

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- Fall 2019
- 2 credit units
- **Rigorous seminar on fundamental and cutting-edge topics in computer architecture**
- Critical presentation, review, and discussion of seminal works in computer architecture
  - We will cover many ideas & issues, analyze their tradeoffs, perform **critical thinking** and brainstorming
- Participation, presentation, synthesis report
- You can register for the course online
- [https://safari.ethz.ch/architecture\\_seminar/fall2018/doku.php](https://safari.ethz.ch/architecture_seminar/fall2018/doku.php)

# Announcement

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- If you are interested in **learning more** and **doing research** in Computer Architecture, three suggestions:
  - **Email me** with your interest (CC: Juan)
  - Take **the seminar course** and the “**Computer Architecture**” course
  - **Do readings** and assignments on your own
- There are **many exciting projects and research positions** available, spanning:
  - Memory systems
  - Hardware security
  - GPUs, FPGAs, heterogeneous systems, ...
  - New execution paradigms (e.g., in-memory computing)
  - Security-architecture-reliability-energy-performance interactions
  - Architectures for medical/health/genomics

# We Are **Almost** Done With This...

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- Single-cycle Microarchitectures
- Multi-cycle and Microprogrammed Microarchitectures
- Pipelining
- Issues in Pipelining: Control & Data Dependence Handling, State Maintenance and Recovery, ...
- Out-of-Order Execution
- Other Execution Paradigms

# Approaches to (Instruction-Level) Concurrency

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- Pipelining
- Out-of-order execution
- Dataflow (at the ISA level)
- Superscalar Execution
- VLIW
- Fine-Grained Multithreading
- Systolic Arrays
- Decoupled Access Execute
- SIMD Processing (Vector and array processors, GPUs)

# Readings for Today

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- Required

- H. T. Kung, “[Why Systolic Architectures?](#),” IEEE Computer 1982.

- Recommended

- Jouppi et al., “[In-Datacenter Performance Analysis of a Tensor Processing Unit](#)”, ISCA 2017.

# Systolic Arrays

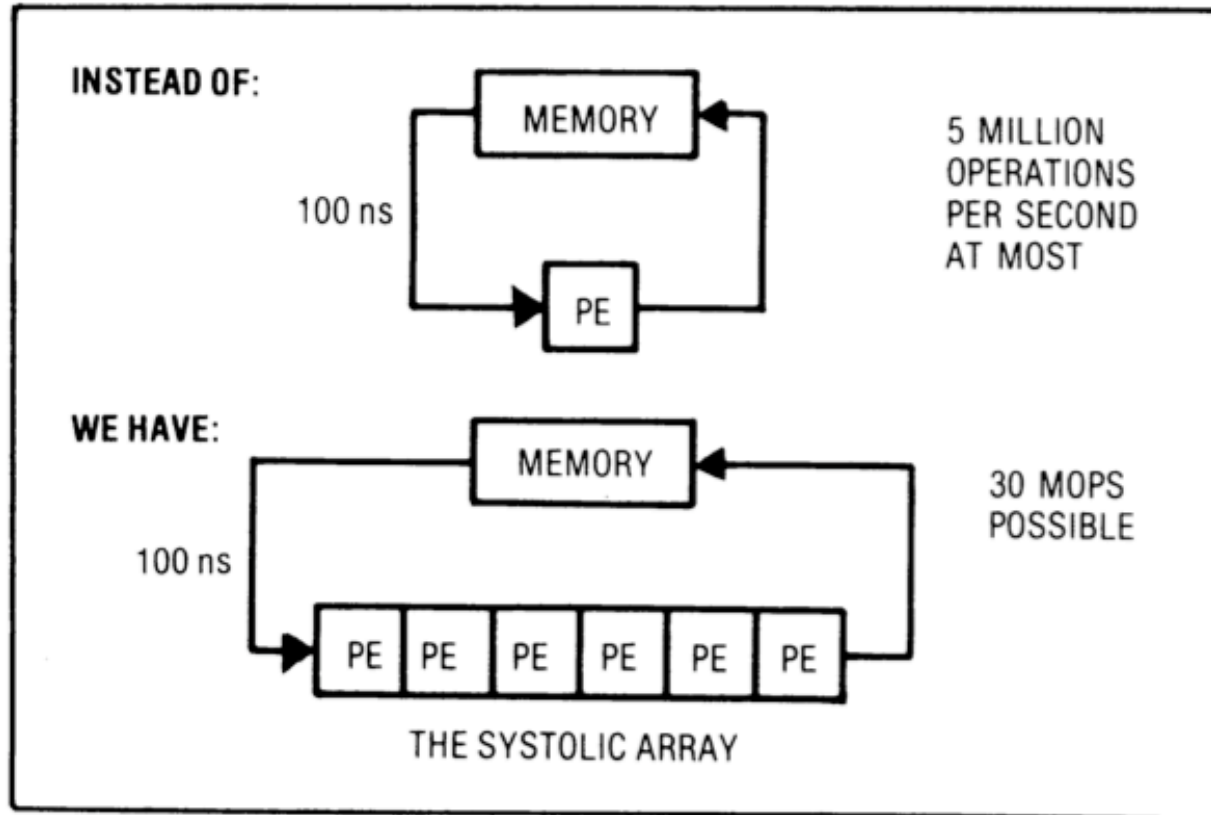
# Systolic Arrays: Motivation

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- Goal: design an accelerator that has
  - Simple, regular design (keep # unique parts small and regular)
  - High concurrency → high performance
  - Balanced computation and I/O (memory) bandwidth
- Idea: Replace a single processing element (PE) with a regular array of PEs and carefully orchestrate flow of data between the PEs
  - such that they collectively transform a piece of input data before outputting it to memory
- Benefit: Maximizes computation done on a single piece of data element brought from memory



# Systolic Arrays



Memory: heart  
Data: blood  
PEs: cells

Memory pulses  
data through  
PEs

Figure 1. Basic principle of a systolic system.

- H. T. Kung, "[Why Systolic Architectures?](#)," IEEE Computer 1982.

# Why Systolic Architectures?

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- Idea: Data flows from the computer memory in a rhythmic fashion, passing through many processing elements before it returns to memory
- Similar to blood flow: heart → many cells → heart
  - Different cells “process” the blood
  - Many veins operate simultaneously
  - Can be many-dimensional
- Why? Special purpose accelerators/architectures need
  - Simple, regular design (keep # unique parts small and regular)
  - High concurrency → high performance
  - Balanced computation and I/O (memory) bandwidth

# Systolic Architectures

- Basic principle: Replace a single PE with a **regular array of PEs** and **carefully orchestrate flow of data** between the PEs
  - Balance computation and memory bandwidth

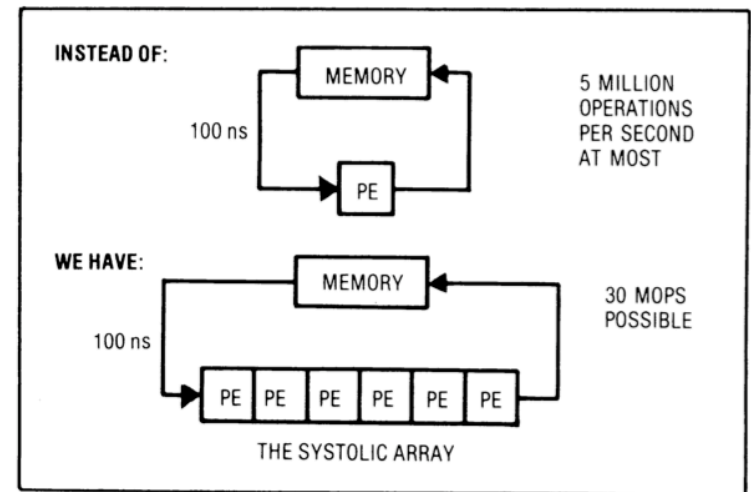


Figure 1. Basic principle of a systolic system.

- Differences from pipelining:
  - These are individual PEs
  - Array structure can be non-linear and multi-dimensional
  - PE connections can be multidirectional (and different speed)
  - PEs can have local memory and execute kernels (rather than a piece of the instruction)

# Systolic Computation Example

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## ■ Convolution

- Used in filtering, pattern matching, correlation, polynomial evaluation, etc ...
- Many **image processing** tasks
- **Machine learning**: up to hundreds of **convolutional layers** in Convolutional Neural Networks (CNN)

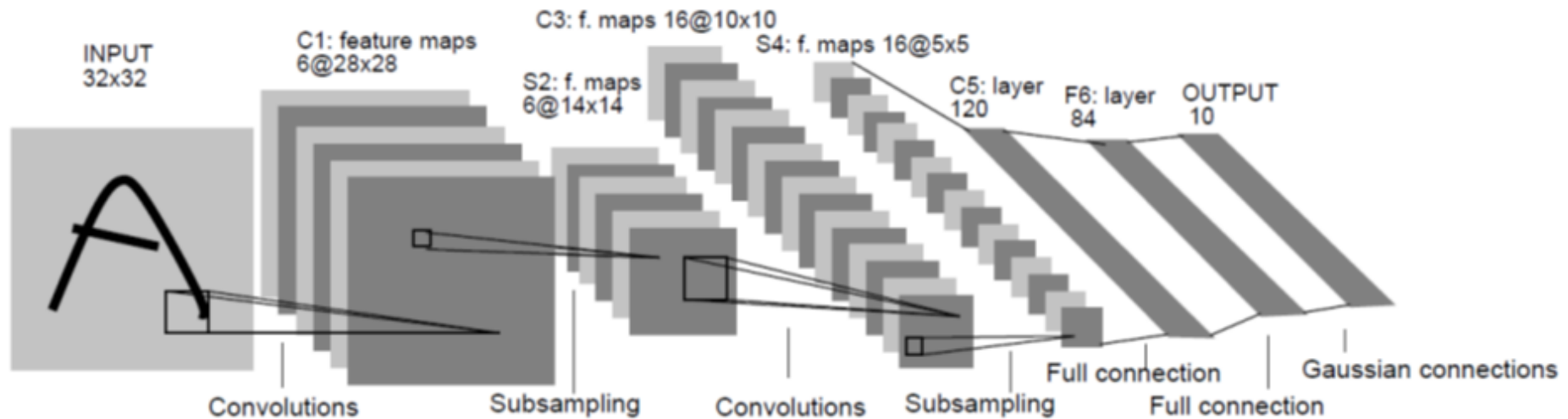
**Given** the sequence of weights  $\{w_1, w_2, \dots, w_k\}$   
and the input sequence  $\{x_1, x_2, \dots, x_n\}$ ,

**compute** the result sequence  $\{y_1, y_2, \dots, y_{n+1-k}\}$   
defined by

$$y_i = w_1x_i + w_2x_{i+1} + \dots + w_kx_{i+k-1}$$

# LeNet-5, a Convolutional Neural Network for Hand-Written Digit Recognition

This is a 1024\*8 bit input, which will have a truth table of  $2^{8196}$  entries



# Convolutional Neural Networks: Demo

[Back to Yann's Home Publications](#)

## LeNet-5 Demos

### Unusual Patterns

[unusual styles](#)  
[weirdos](#)

### Invariance

[translation](#) (anim)  
[scale](#) (anim)  
[rotation](#) (anim)  
[squeezing](#) (anim)  
[stroke width](#) (anim)

### Noise Resistance

[noisy 3 and 6](#)  
[noisy 2](#) (anim)  
[noisy 4](#) (anim)

### Multiple Character

[various stills](#)  
[dancing 00](#) (anim)  
[dancing 384](#) (anim)

### Complex cases

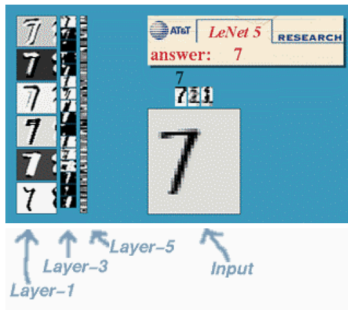
(anim)  
[35 -> 53](#)  
[12 -> 4 -> 21](#)  
[23 -> 32](#)  
[30 + noise](#)  
[31-51-57-61](#)

## LeNet-5, convolutional neural networks

Convolutional Neural Networks are a special kind of multi-layer neural networks. Like almost every other neural networks they are trained with a version of the back-propagation algorithm. Where they differ is in the architecture.

Convolutional Neural Networks are designed to recognize visual patterns directly from pixel images with minimal preprocessing. They can recognize patterns with extreme variability (such as handwritten characters), and with robustness to distortions and simple geometric transformations.

LeNet-5 is our latest convolutional network designed for handwritten and machine-printed character recognition. Here is an example of LeNet-5 in action.



Many more examples are available in the column on the left:

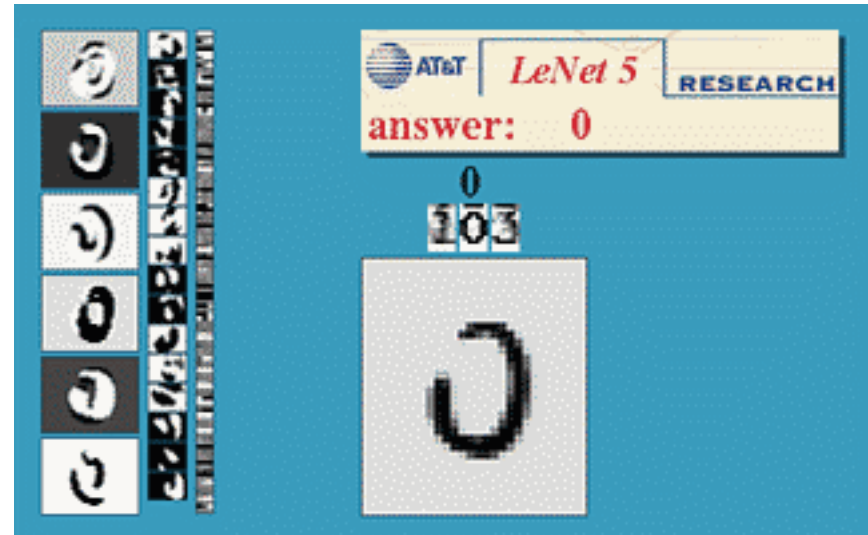
Several papers on LeNet and convolutional networks are available on my [publication page](#):

[LeCun et al., 1998]

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner.  
Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, november 1998.  
[ps.gz](#)

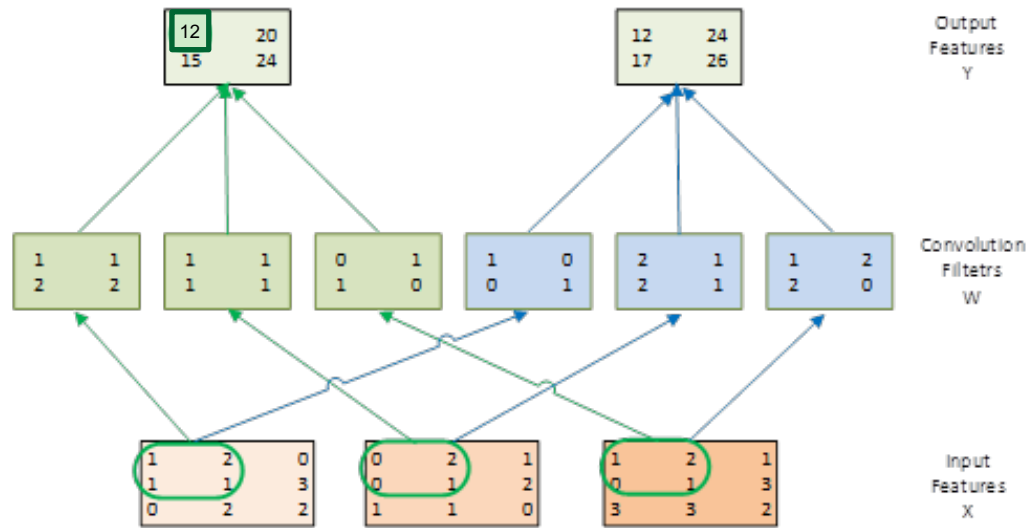
[Bottou et al., 1997]

L. Bottou, Y. LeCun, and Y. Bengio. Global training of



<http://yann.lecun.com/exdb/lenet/index.html>

# Implementing a Convolutional Layer with Matrix Multiplication



$$\begin{array}{|c|c|c|} \hline 1 & 1 & 2 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 1 \\ \hline \end{array} \quad
 \begin{array}{|c|c|c|} \hline 2 & 2 & 2 \\ \hline 1 & 1 & 1 \\ \hline 2 & 2 & 0 \\ \hline \end{array} \quad
 \begin{array}{|c|c|c|} \hline 0 & 1 & 1 \\ \hline 1 & 2 & 2 \\ \hline 1 & 2 & 0 \\ \hline \end{array}
 \quad * \quad
 \begin{array}{|c|c|c|c|} \hline 1 & 2 & 1 & 1 \\ \hline 2 & 0 & 1 & 3 \\ \hline 1 & 1 & 0 & 2 \\ \hline 1 & 3 & 2 & 2 \\ \hline 0 & 2 & 0 & 1 \\ \hline 2 & 1 & 1 & 2 \\ \hline 0 & 1 & 1 & 1 \\ \hline 1 & 2 & 1 & 0 \\ \hline 1 & 2 & 1 & 1 \\ \hline 2 & 1 & 0 & 3 \\ \hline 0 & 1 & 3 & 3 \\ \hline 1 & 3 & 3 & 2 \\ \hline \end{array}
 \quad = \quad
 \begin{array}{|c|c|c|c|} \hline 14 & 20 & 15 & 24 \\ \hline 12 & 24 & 17 & 26 \\ \hline \end{array}$$

Convolution Filters  $W'$

Input Features  $X_{\text{unrolled}}$

Output Features  $Y$

# Power of **Convolutions** and **Applied Courses**

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- In 2010, Prof. Andreas Moshovos adopted Professor Hwu's ECE498AL Programming Massively Parallel Processors Class
- Several of Prof. Geoffrey Hinton's graduate students took the course
- These students developed the GPU implementation of the Deep CNN that was trained with 1.2M images to win the ImageNet competition



# Example: AlexNet (2012)

- AlexNet won **ImageNet** with more than 10.8% points ahead of the runner up
  - Krizhevsky et al., “**ImageNet Classification with Deep Convolutional Neural Networks**”, NIPS 2012.

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## ImageNet Classification with Deep Convolutional Neural Networks

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### Abstract

We trained a large, deep convolutional neural network to classify the 1.2 million high-resolution images in the ImageNet LSVRC-2010 contest into the 1000 different classes. On the test data, we achieved top-1 and top-5 error rates of 37.5% and 17.0% which is considerably better than the previous state-of-the-art. The neural network, which has 60 million parameters and 650,000 neurons, consists of five convolutional layers, some of which are followed by max-pooling layers, and three fully-connected layers with a final 1000-way softmax. To make training faster, we used non-saturating neurons and a very efficient GPU implementation of the convolution operation. To reduce overfitting in the fully-connected layers we employed a recently-developed regularization method called “dropout” that proved to be very effective. We also entered a variant of this model in the ILSVRC-2012 competition and achieved a winning top-5 test error rate of 15.3%, compared to 26.2% achieved by the second-best entry.

# Example: GoogLeNet (2014)

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- Google improves the precision by adding more layers
  - From 8 in AlexNet to 22 in GoogLeNet
  - Szegedy et al., “Going Deeper with Convolutions”, CVPR 2015.

## Going Deeper with Convolutions

Christian Szegedy<sup>1</sup>, Wei Liu<sup>2</sup>, Yangqing Jia<sup>1</sup>, Pierre Sermanet<sup>1</sup>, Scott Reed<sup>3</sup>,  
Dragomir Anguelov<sup>1</sup>, Dumitru Erhan<sup>1</sup>, Vincent Vanhoucke<sup>1</sup>, Andrew Rabinovich<sup>4</sup>

<sup>1</sup>Google Inc. <sup>2</sup>University of North Carolina, Chapel Hill

<sup>3</sup>University of Michigan, Ann Arbor <sup>4</sup>Magic Leap Inc.

<sup>1</sup>{szegedy, jia, yq, sermanet, dragomir, dimitru, vanhoucke}@google.com

<sup>2</sup>wliu@cs.unc.edu, <sup>3</sup>reedscott@umich.edu, <sup>4</sup>arabinovich@magic Leap.com

# Example: ResNet (2015)

- He et al., “Deep Residual Learning for Image Recognition”, CVPR 2016.

## Deep Residual Learning for Image Recognition

Kaiming He

Xiangyu Zhang

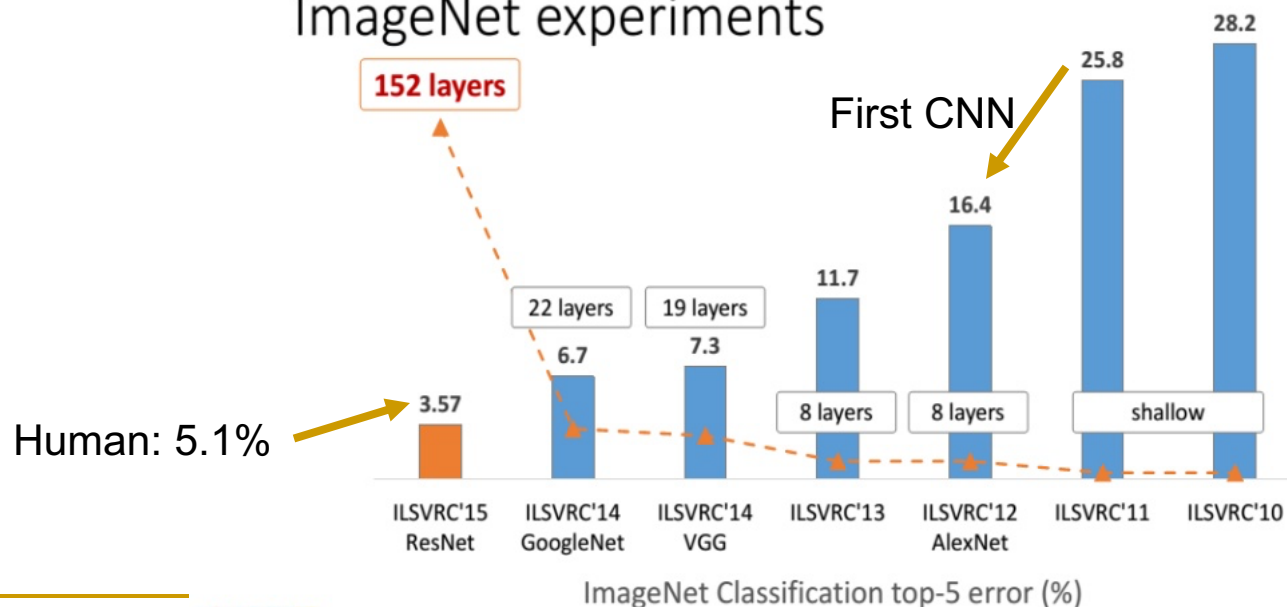
Shaoqing Ren

Jian Sun

Microsoft Research

{kahe, v-xiangz, v-shren, jiansun}@microsoft.com

### ImageNet experiments



# Systolic Computation Example: Convolution (I)

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## ■ Convolution

- Used in filtering, pattern matching, correlation, polynomial evaluation, etc ...
- Many **image processing** tasks
- **Machine learning**: up to hundreds of **convolutional layers** in Convolutional Neural Networks (CNN)

**Given** the sequence of weights  $\{w_1, w_2, \dots, w_k\}$   
and the input sequence  $\{x_1, x_2, \dots, x_n\}$ ,

**compute** the result sequence  $\{y_1, y_2, \dots, y_{n+1-k}\}$   
defined by

$$y_i = w_1x_i + w_2x_{i+1} + \dots + w_kx_{i+k-1}$$

# Systolic Computation Example: Convolution (II)

- $y_1 = w_1x_1 + w_2x_2 + w_3x_3$
- $y_2 = w_1x_2 + w_2x_3 + w_3x_4$
- $y_3 = w_1x_3 + w_2x_4 + w_3x_5$

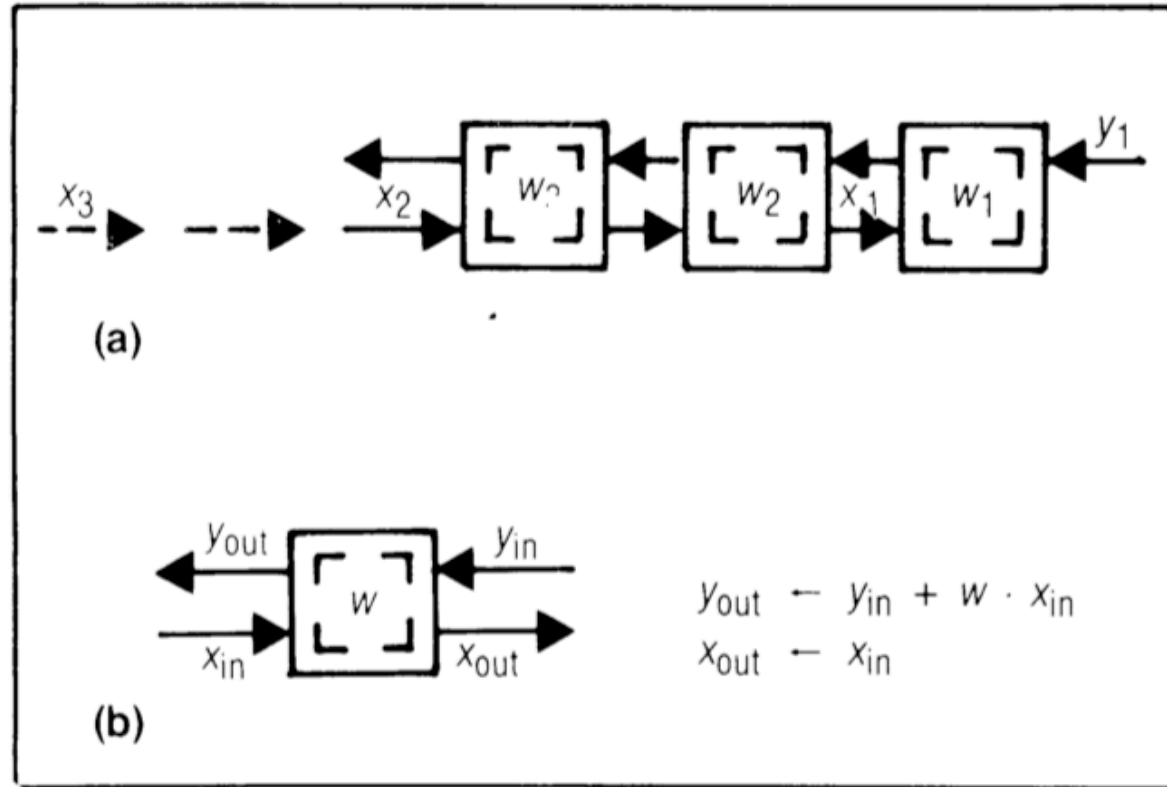


Figure 8. Design W1: systolic convolution array (a) and cell (b) where  $w_i$ 's stay and  $x_i$ 's and  $y_i$ 's move systolically in opposite directions.

# Systolic Computation Example: Convolution (III)

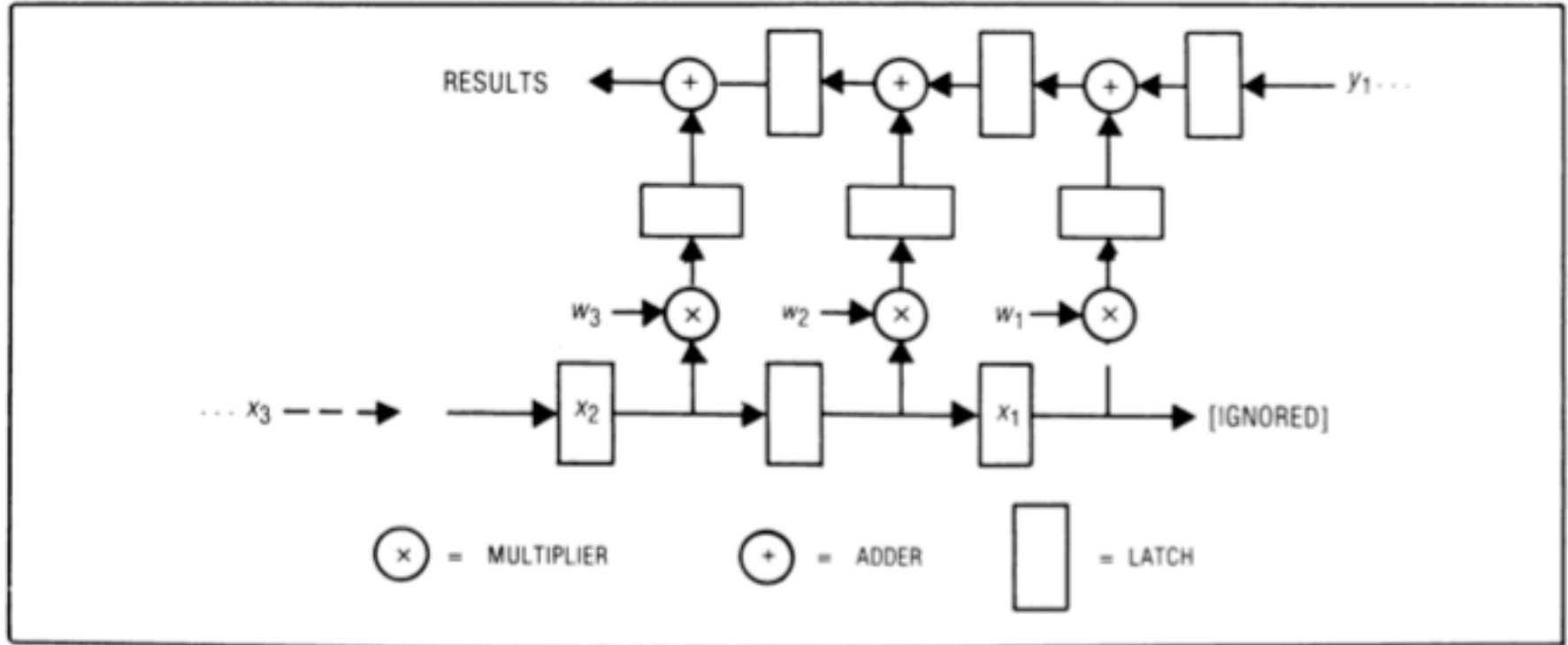


Figure 10. Overlapping the executions of multiply and add in design W1.

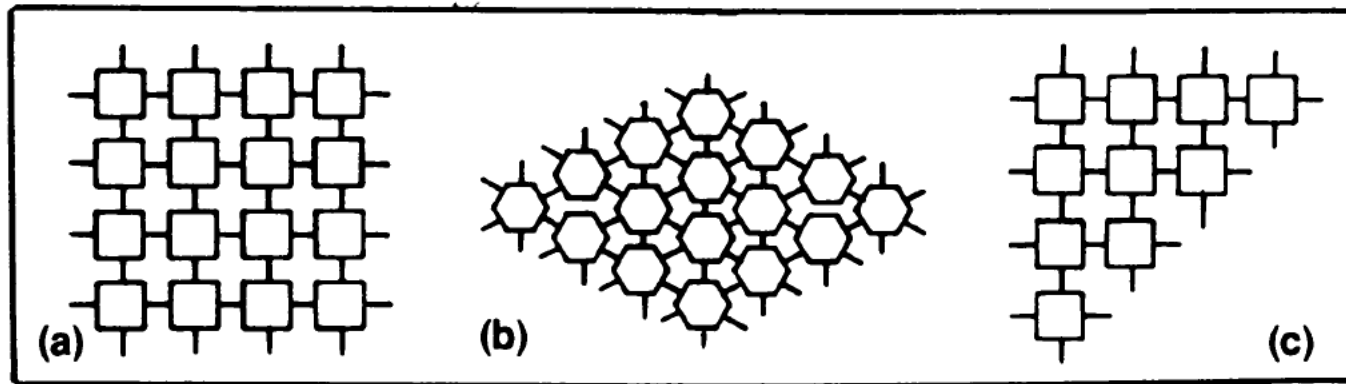
- Worthwhile to implement adder and multiplier separately to allow overlapping of add/mul executions

# Systolic Computation Example: Convolution (IV)

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- One needs to **carefully orchestrate** when **data elements** are **input to the array**
- And when **output is buffered**
- This gets more involved when
  - Array dimensionality increases
  - PEs are less predictable in terms of latency

# Two-Dimensional Systolic Arrays



**Figure 11. Two-dimensional systolic arrays: (a) type R, (b) type H, and (c) type T.**

To a given problem there could be both one- and two-dimensional systolic array solutions. For example, two-dimensional convolution can be performed by a one-dimensional systolic array<sup>24,25</sup> or a two-dimensional systolic array.<sup>6</sup> When the memory speed is more than cell speed, two-dimensional systolic arrays such as those depicted in Figure 11 should be used. At each cell cycle, all the I/O ports on the array boundaries can input or output data items to or from the memory; as a result, the available memory bandwidth can be fully utilized. Thus, the choice of a one- or two-dimensional scheme is very dependent on how cells and memories will be implemented.



# Combinations

- Systolic arrays can be chained together to form powerful systems
- This systolic array is capable of producing on-the-fly least-squares fit to all the data that has arrived up to any given moment

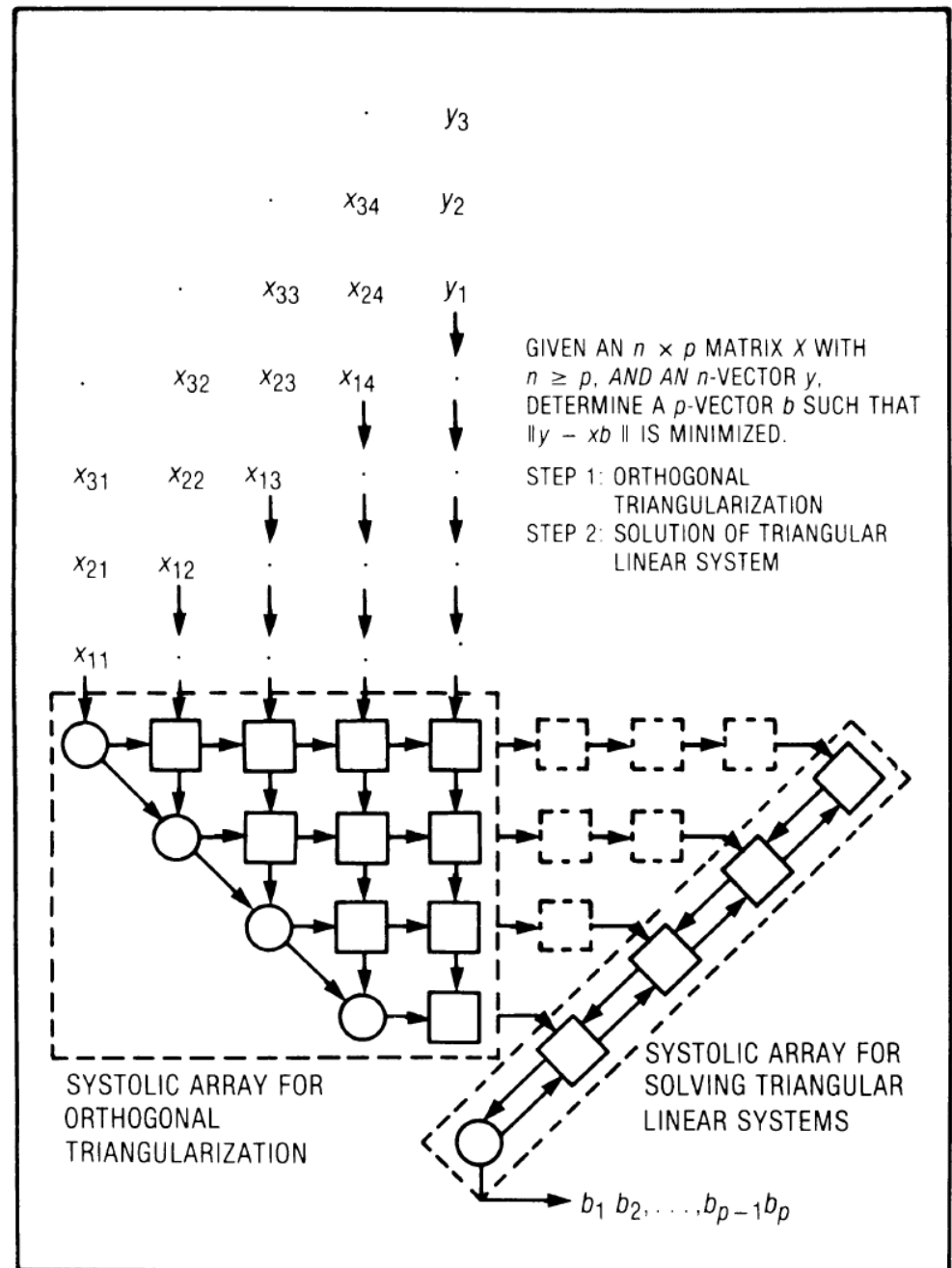


Figure 12. On-the-fly least-squares solutions using one- and two-dimensional systolic arrays, with  $p = 4$ .

# Systolic Arrays: Pros and Cons

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## ■ Advantages:

- ❑ Principled: Efficiently makes use of limited memory bandwidth, balances computation to I/O bandwidth availability
- ❑ Specialized (computation needs to fit PE organization/functions)
  - improved efficiency, simple design, high concurrency/performance
  - good to do more with less memory bandwidth requirement

## ■ Downside:

- ❑ Specialized
  - **not generally applicable** because computation needs to fit the PE functions/organization

# More Programmability in Systolic Arrays

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- Each PE in a systolic array
  - Can store multiple “weights”
  - Weights can be selected on the fly
  - Eases implementation of, e.g., adaptive filtering
- Taken further
  - Each PE can have its own data and instruction memory
  - Data memory → to store partial/temporary results, constants
  - Leads to **stream processing, pipeline parallelism**
    - More generally, **staged execution**

# Pipeline-Parallel (Pipelined) Programs

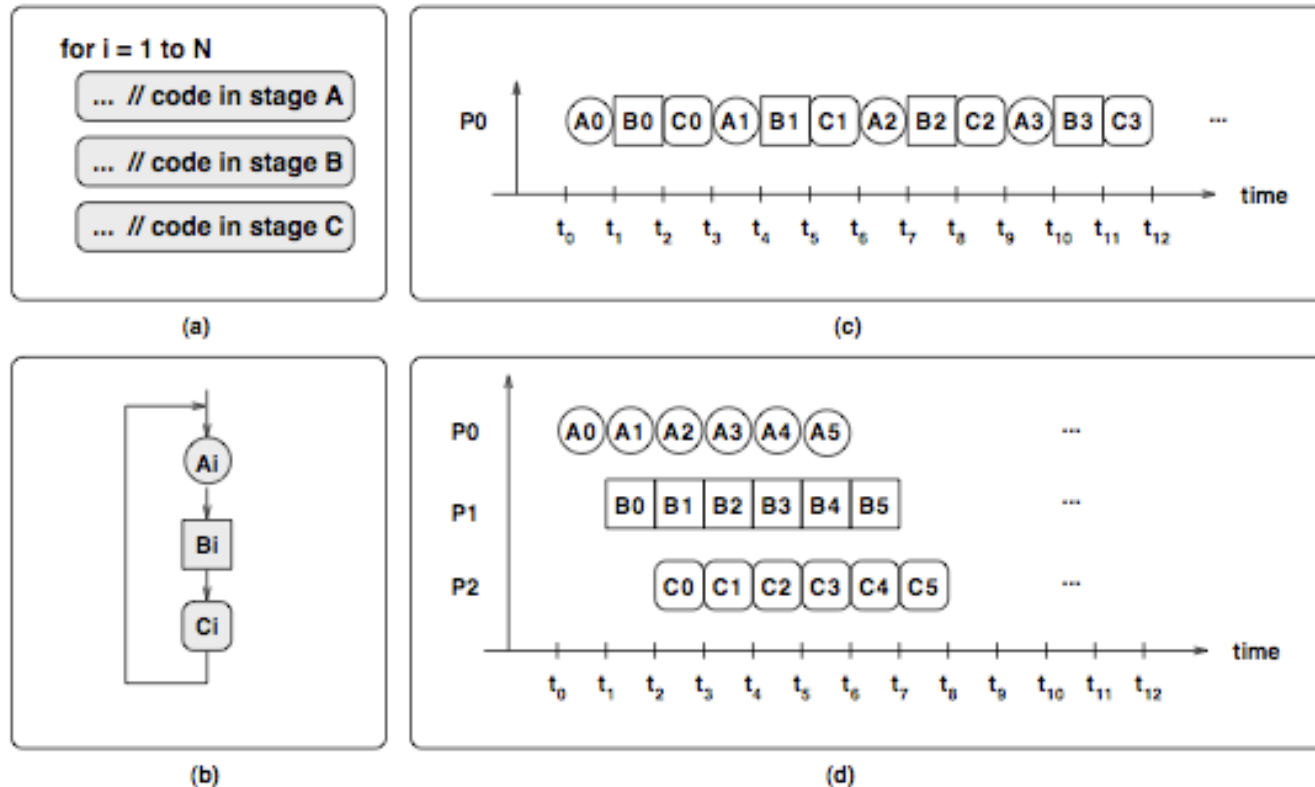
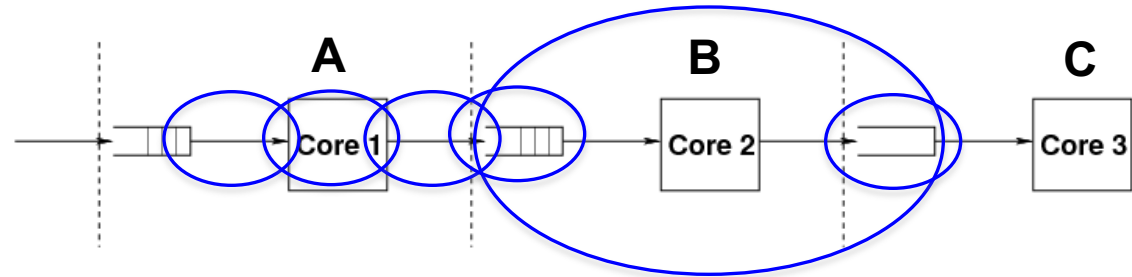


Figure 1. (a) The code of a loop, (b) Each iteration is split into 3 pipeline stages: A, B, and C. Iteration  $i$  comprises  $A_i$ ,  $B_i$ ,  $C_i$ . (c) Sequential execution of 4 iterations. (d) Parallel execution of 6 iterations using pipeline parallelism on a three-core machine. Each stage executes on one core.

# Stages of Pipelined Programs

- Loop iterations are divided into **code segments called stages**
- Threads execute stages on different cores

```
loop {  
  Compute1  A  
  Compute2  B  
  Compute3  C  
}
```



# Pipelined File Compression Example

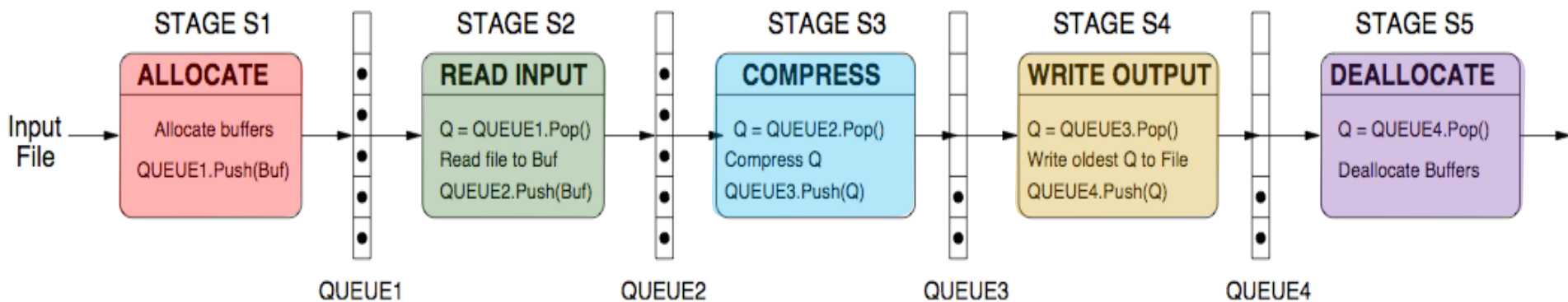


Figure 3. File compression algorithm executed using pipeline parallelism

# Systolic Array: Advantages & Disadvantages

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## ■ Advantages

- ❑ Makes **multiple uses of each data item** → reduced need for fetching/refetching → better use of memory bandwidth
- ❑ **High concurrency**
- ❑ Regular design (both data and control flow)

## ■ Disadvantages

- ❑ **Not good at exploiting irregular parallelism**
- ❑ Relatively special purpose → need software, programmer support to be a general purpose model

# Example Systolic Array: The WARP Computer

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- HT Kung, CMU, 1984-1988
- Linear array of 10 cells, each cell a 10 Mflop programmable processor
- Attached to a general purpose host machine
- HLL and optimizing compiler to program the systolic array
- Used extensively to accelerate vision and robotics tasks
- Annaratone et al., “Warp Architecture and Implementation,” ISCA 1986.
- Annaratone et al., “The Warp Computer: Architecture, Implementation, and Performance,” IEEE TC 1987.



# The WARP Computer

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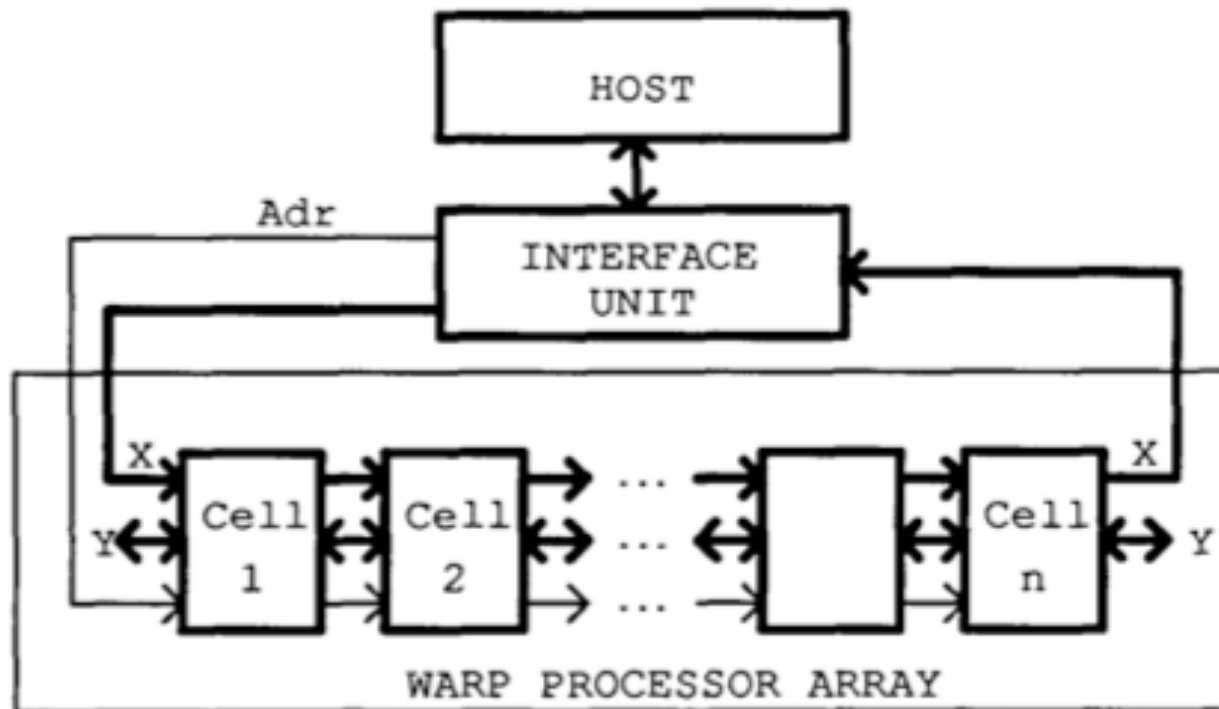


Figure 1: Warp system overview

# The WARP Cell

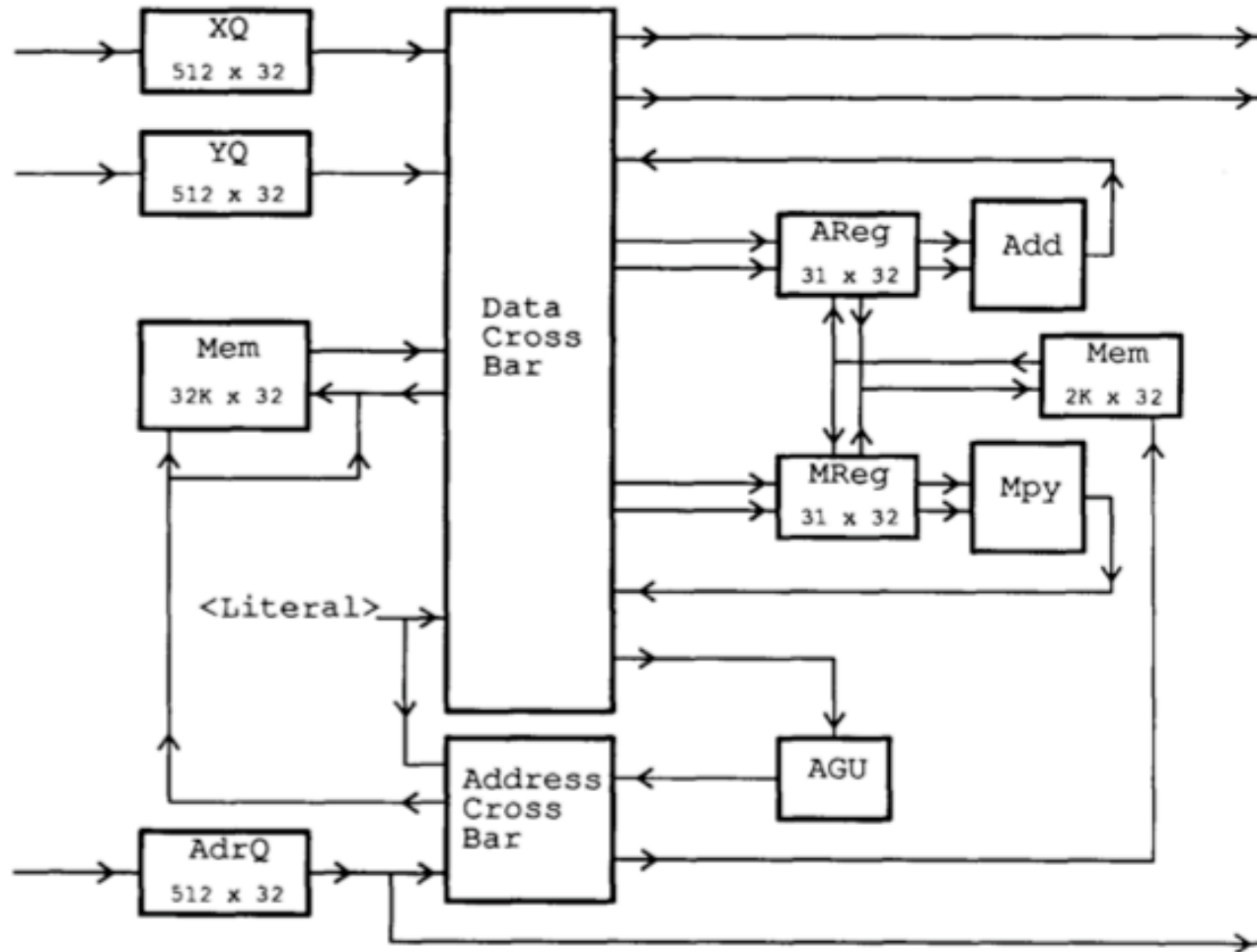
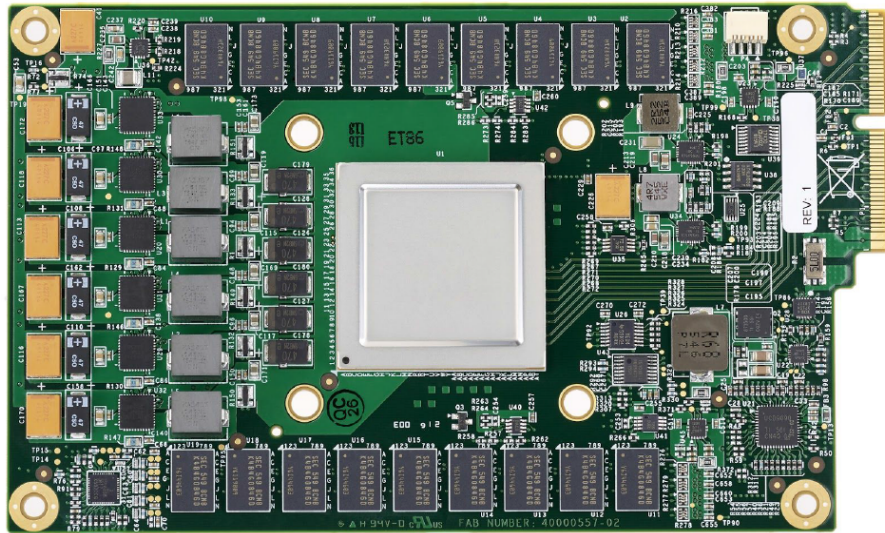
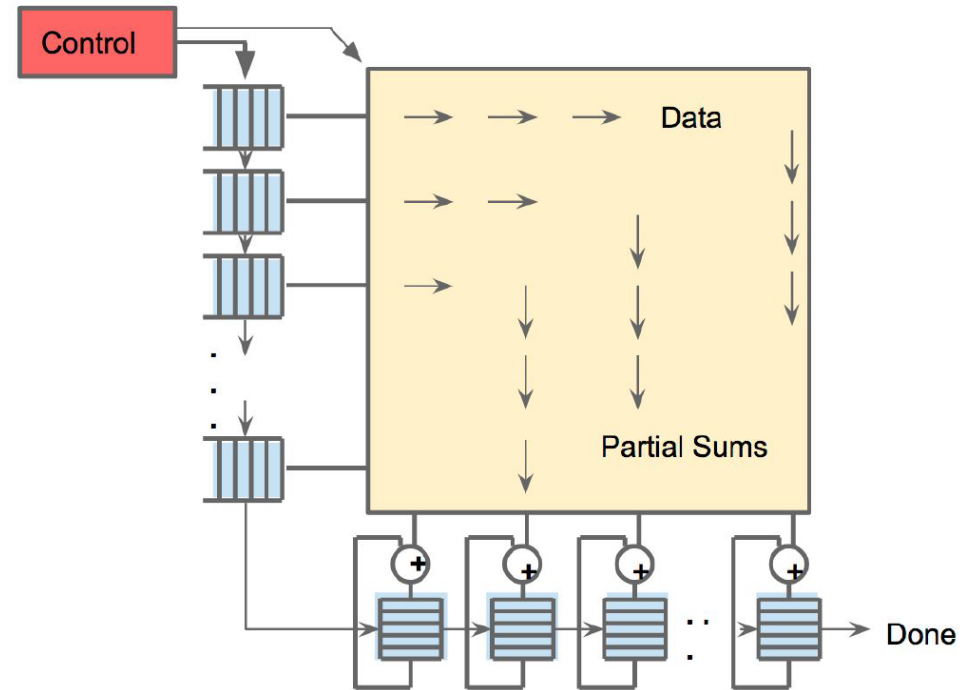


Figure 2: Warp cell data path

# An Example Modern Systolic Array: TPU (I)



**Figure 3.** TPU Printed Circuit Board. It can be inserted in the slot for an SATA disk in a server, but the card uses PCIe Gen3 x16.

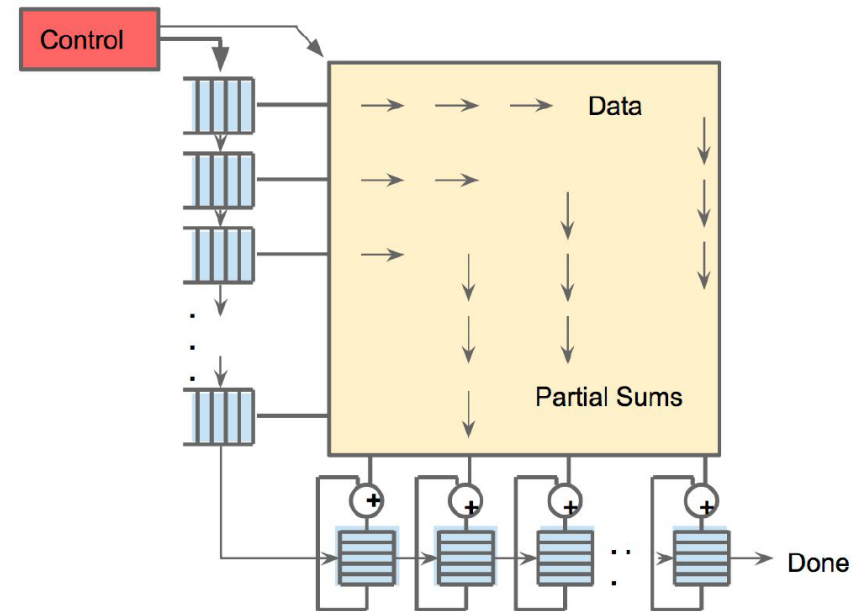


**Figure 4.** Systolic data flow of the Matrix Multiply Unit. Software has the illusion that each 256B input is read at once, and they instantly update one location of each of 256 accumulator RAMs.

Jouppi et al., “In-Datcenter Performance Analysis of a Tensor Processing Unit”, ISCA 2017.

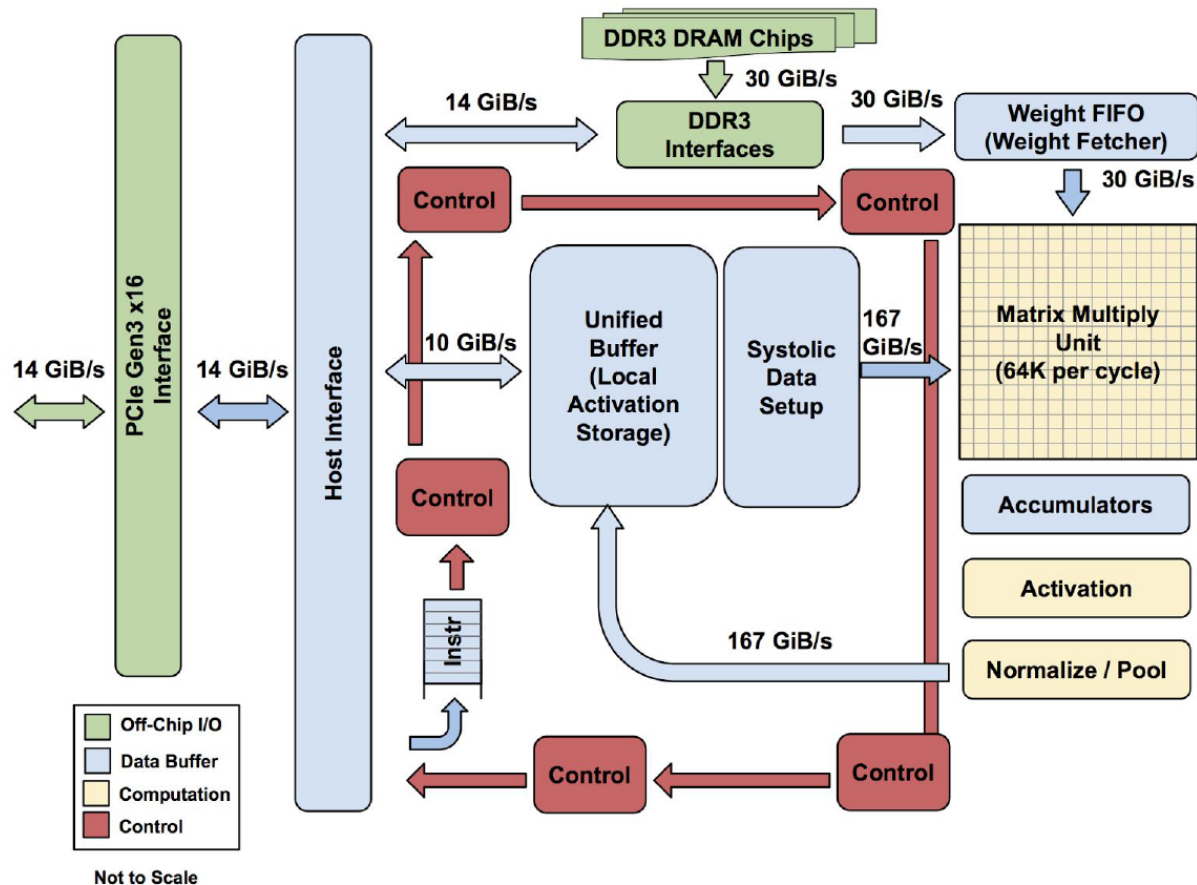
# An Example Modern Systolic Array: TPU (II)

As reading a large SRAM uses much more power than arithmetic, the matrix unit uses systolic execution to save energy by reducing reads and writes of the Unified Buffer [Kun80][Ram91][Ovt15b]. Figure 4 shows that data flows in from the left, and the weights are loaded from the top. A given 256-element multiply-accumulate operation moves through the matrix as a diagonal wavefront. The weights are preloaded, and take effect with the advancing wave alongside the first data of a new block. Control and data are pipelined to give the illusion that the 256 inputs are read at once, and that they instantly update one location of each of 256 accumulators. From a correctness perspective, software is unaware of the systolic nature of the matrix unit, but for performance, it does worry about the latency of the unit.



Jouppi et al., “In-Datacenter Performance Analysis of a Tensor Processing Unit”, ISCA 2017.

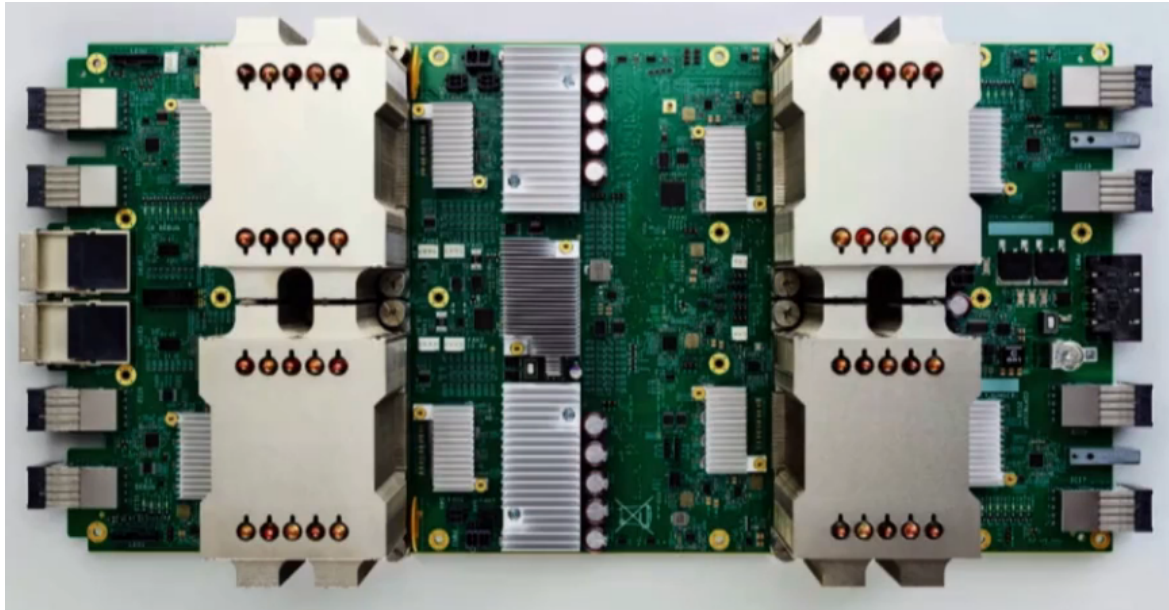
# An Example Modern Systolic Array: TPU (III)



**Figure 1.** TPU Block Diagram. The main computation part is the yellow Matrix Multiply unit in the upper right hand corner. Its inputs are the blue Weight FIFO and the blue Unified Buffer (UB) and its output is the blue Accumulators (Acc). The yellow Activation Unit performs the nonlinear functions on the Acc, which go to the UB.

# TPU: Second Generation

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<https://www.nextplatform.com/2017/05/17/first-depth-look-googles-new-second-generation-tpu/>

4 TPU chips  
vs 1 chip in TPU1

High Bandwidth Memory  
vs DDR3

Floating point operations  
vs FP16

45 TFLOPS per chip  
vs 23 TOPS

Designed for training  
and inference  
vs only inference

# Approaches to (Instruction-Level) Concurrency

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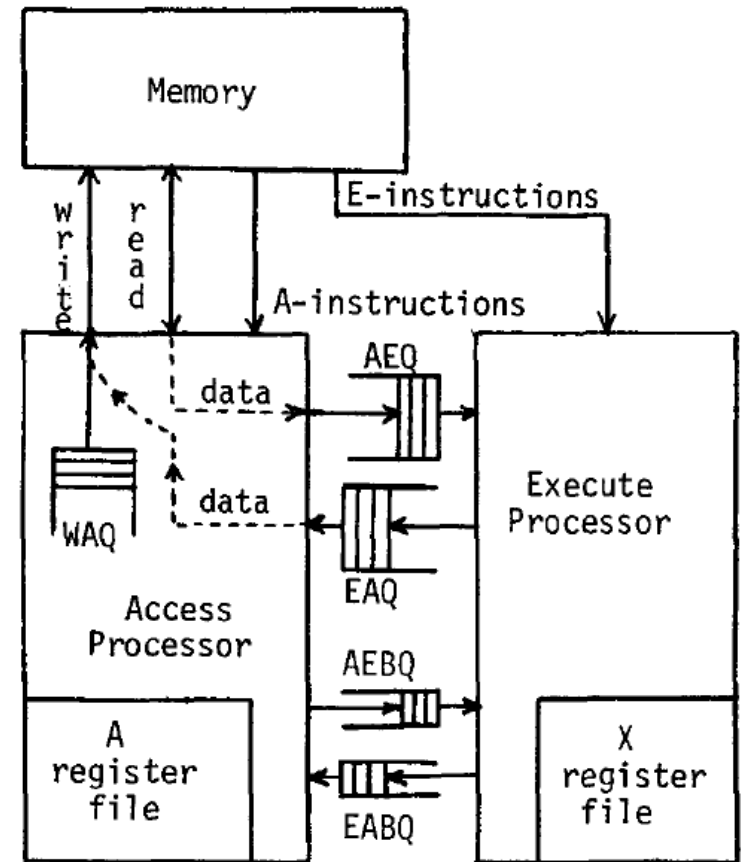
- Pipelining
- Out-of-order execution
- Dataflow (at the ISA level)
- Superscalar Execution
- VLIW
- Fine-Grained Multithreading
- Systolic Arrays
- Decoupled Access Execute
- SIMD Processing (Vector and array processors, GPUs)

# Decoupled Access/Execute (DAE)



# Decoupled Access/Execute (DAE)

- Motivation: **Tomasulo's algorithm too complex** to implement
  - 1980s before Pentium Pro
- Idea: Decouple operand access and execution via **two separate instruction streams that communicate via ISA-visible queues**.
- Smith, “[Decoupled Access/Execute Computer Architectures](#),” ISCA 1982, ACM TOCS 1984.



# Decoupled Access/Execute (II)

- Compiler generates two instruction streams (A and E)
  - Synchronizes the two upon control flow instructions (using branch queues)

```

q = 0.0
Do 1 k = 1, 400
1  x(k) = q + y(k) * (r * z(k+10) + t * z(k+11))
    
```

Fig. 2a. Lawrence Livermore Loop 1 (HYDRO EXCERPT)

<pre> loop:  A7 ← -400         A2 ← 0         A3 ← 1         X2 ← r         X5 ← t         X3 ← z + 10, A2         X7 ← z + 11, A2         X4 ← X2 *f X3         X3 ← X5 *f X7         X7 ← y, A2         X6 ← X3 +f X4         X4 ← X7 *f X6         A7 ← A7 + 1         x, A2 ← X4         A2 ← A2 + A3         JAM loop     </pre>	<pre> . negative loop count . initialize index . index increment . load loop invariants . into registers . load z(k+10) . load z(k+11) . r*z(k+10)-flt. mult. . t * z(k+11) . load y(k) . r*z(x+10)+t*z(k+11)) . y(k) * (above) . increment loop counter . store into x(k) . increment index . Branch if A7 &lt; 0     </pre>
---	---

Fig. 2b. Compilation onto CRAY-1-like architecture

<u>Access</u>	<u>Execute</u>
.	
.	
.	
AEQ ← z + 10, A2	X4 ← X2 *f AEQ
AEQ ← z + 11, A2	X3 ← X5 *f AEQ
AEQ ← y, A2	X6 ← X3 +f X4
A7 ← A7 + 1	EAQ ← AEQ *f X6
x, A2 ← EAQ	.
A2 ← A2+ A3	.
.	.
.	
.	

Fig. 2c. Access and execute programs for straight-line section of loop

# Decoupled Access/Execute (III)

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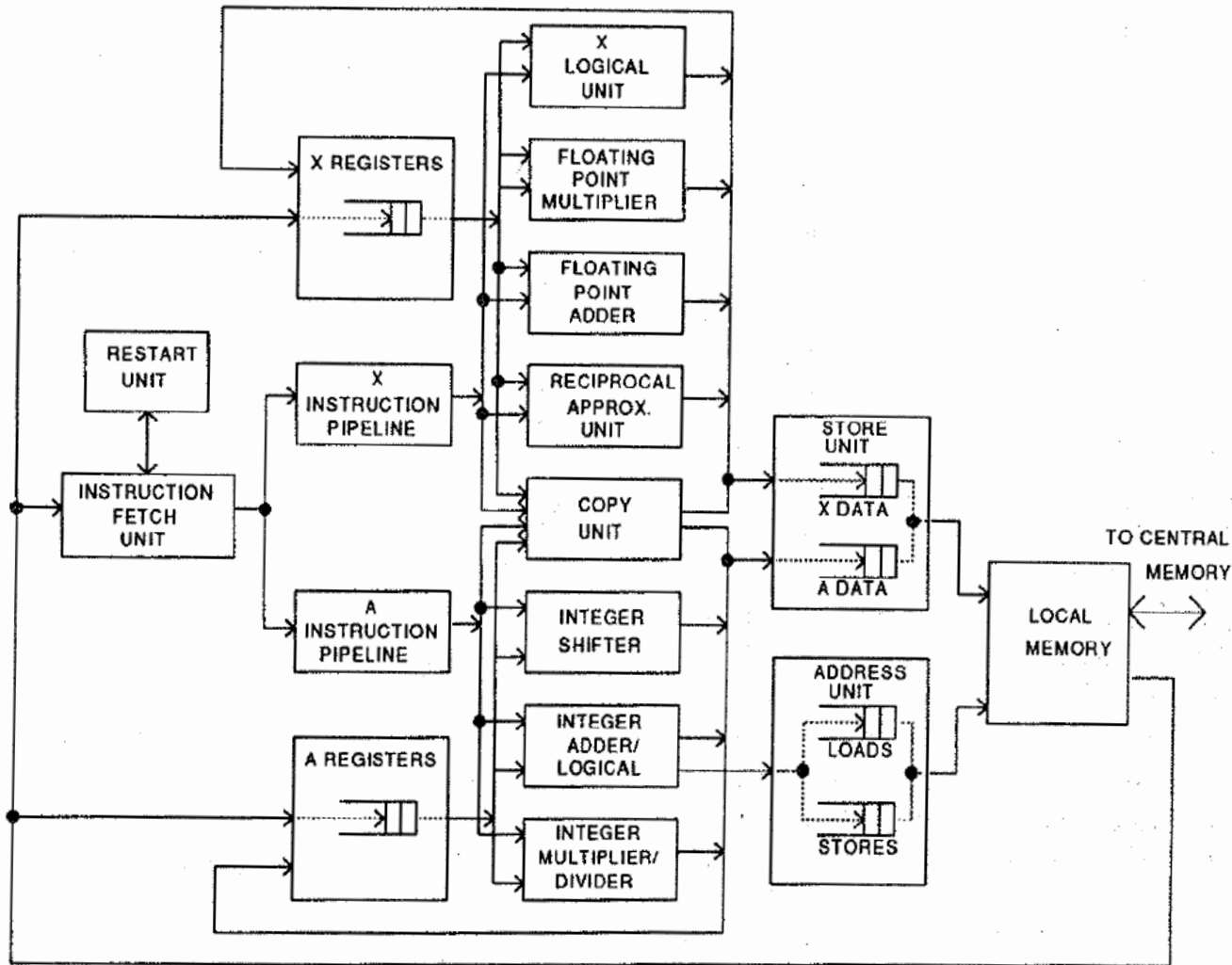
## ■ Advantages:

- + Execute stream can run ahead of the access stream and vice versa
  - + If A is waiting for memory, E can perform useful work
  - + If A hits in cache, it supplies data to lagging E
  - + Queues reduce the number of required registers
- + Limited out-of-order execution without wakeup/select complexity

## ■ Disadvantages:

- **Compiler support** to partition the program and manage queues
  - Determines the amount of decoupling
- Branch instructions require synchronization between A and E
- Multiple instruction streams (can be done with a single one, though)

# Astronautics ZS-1



- Single stream steered into A and X pipelines
- Each pipeline in-order
- Smith et al., “[The ZS-1 central processor](#),” ASPLOS 1987.
- Smith, “[Dynamic Instruction Scheduling and the Astronautics ZS-1](#),” IEEE Computer 1989.

# Loop Unrolling to Eliminate Branches

```
for (int i = 0; i < N; i++){  
  
    A[i] = A[i] + B[i];  
  
}
```

```
for (int i = 0; i < N; i+=4){  
  
    A[i]   = A[i]   + B[i];  
    A[i+1] = A[i+1] + B[i+1];  
    A[i+2] = A[i+2] + B[i+2];  
    A[i+3] = A[i+3] + B[i+3];  
  
}
```

- Idea: Replicate loop body multiple times within an iteration
- + Reduces loop maintenance overhead
  - Induction variable increment or loop condition test
- + Enlarges basic block (and analysis scope)
  - Enables code optimization and scheduling opportunities
- What if iteration count not a multiple of unroll factor? (need extra code to detect this)
- Increases code size

# A Modern DAE Example: Pentium 4

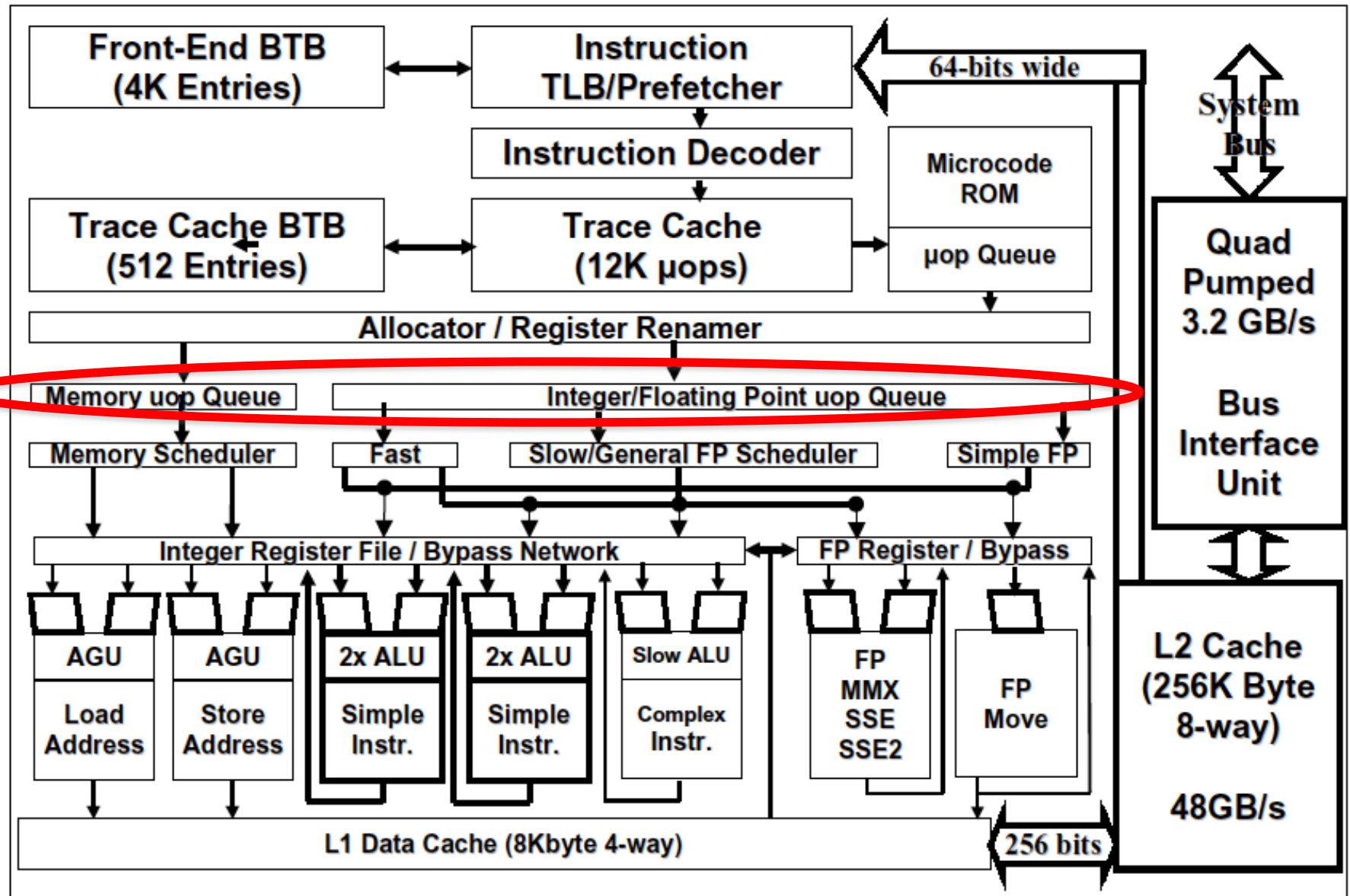
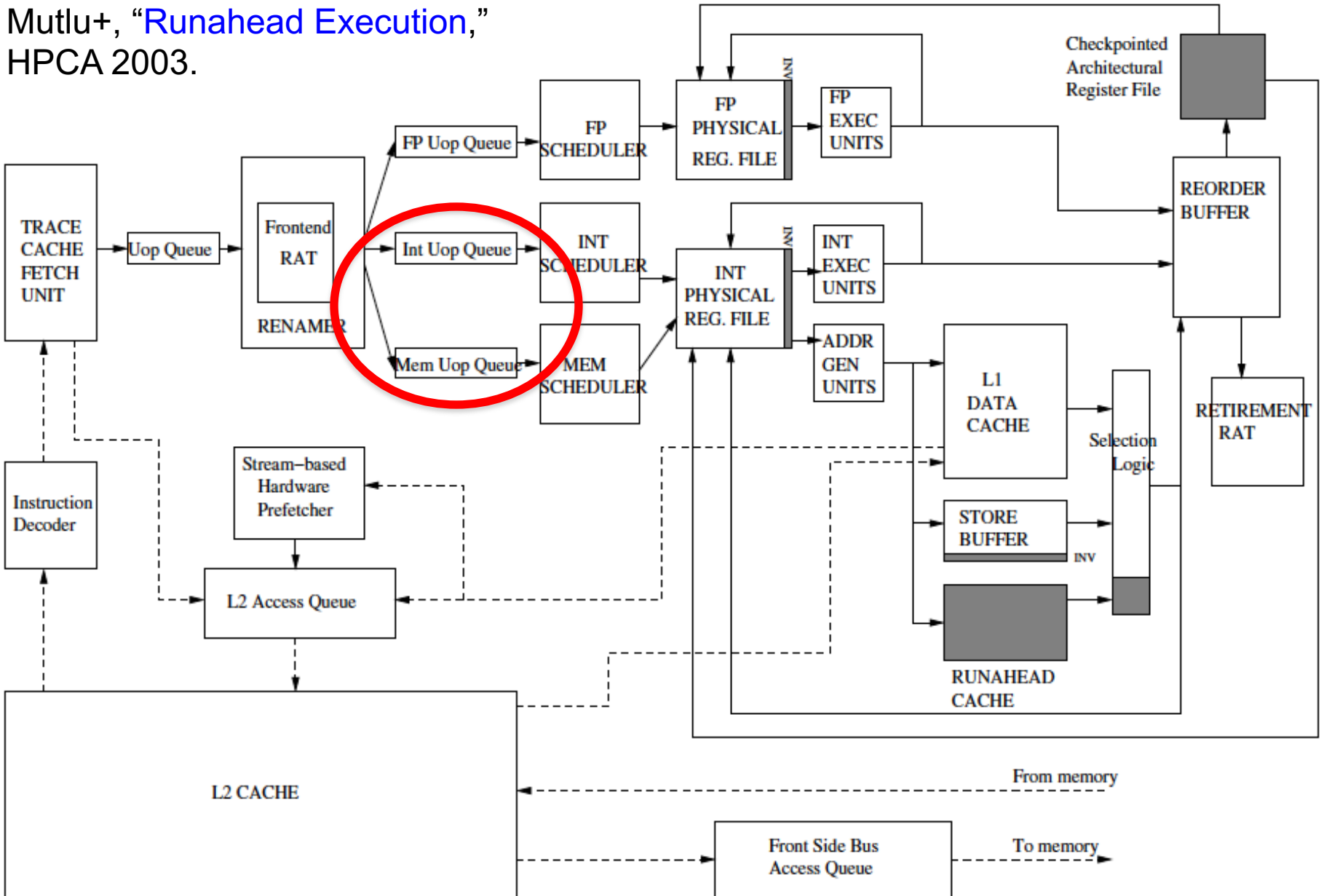


Figure 4: Pentium<sup>®</sup> 4 processor microarchitecture

# Intel Pentium 4 Simplified

Mutlu+, “Runahead Execution,”  
HPCA 2003.



# Approaches to (Instruction-Level) Concurrency

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- Pipelining
- Out-of-order execution
- Dataflow (at the ISA level)
- Superscalar Execution
- VLIW
- Fine-Grained Multithreading
- Systolic Arrays
- Decoupled Access Execute
- SIMD Processing (Vector and array processors, GPUs)



# Design of Digital Circuits

## Lecture 19b: Systolic Arrays and Beyond

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3 May 2019